CH-6 INTEGRATION (SOLUTIONS)

Practice Questions I

(1.) $$I = \int \left(6 - \frac{5}{4}x^2 + 2e^x\right)dx = 6\int dx - \frac{5}{4}\int x^2 dx + 2\int e^x dx$$

$$= 6x - \frac{5}{4}\cdot\frac{x^3}{3} + 2e^x + C$$

$$= 6x - \frac{5}{12}x^3 + 2e^x + C$$

(2.) $$I = \int \frac{x^3 + 2x + 5}{\sqrt{x}}\,dx = \int\left(\frac{x^3}{\sqrt{x}} + 2\frac{x}{\sqrt{x}} + \frac{5}{\sqrt{x}}\right)dx$$

$$= \int\left(x^{3-\frac{1}{2}} + 2x^{1-\frac{1}{2}} + 5x^{-1/2}\right)dx$$

$$= \int\left(x^{5/2} + 2x^{1/2} + 5x^{-1/2}\right)dx$$

$$= \frac{x^{7/2}}{7/2} + 2\cdot\frac{x^{3/2}}{3/2} + 5\frac{x^{1/2}}{1/2} + C$$

$$= \frac{2}{7}x^{7/2} + \frac{4}{3}x^{3/2} + 10x^{1/2} + C$$

(3.) $$I = \int\left(5x + \frac{4}{x} + 3e^x\right)dx = 5\int x\,dx + 4\int\frac{1}{x}dx + 3\int e^x dx$$

$$= 5\frac{x^2}{2} + 4\log x + 3e^x + C$$

$$= \frac{5}{2}x^2 + 4\log x + 3e^x + C.$$

(4.) $$I = \int (x^2 - 5)(x - 3)\,dx = \int (x^3 - 3x^2 - 5x + 15)\,dx$$

$$= \frac{x^4}{4} - \frac{\cancel{3}x^3}{\cancel{3}} - 5\frac{x^2}{2} + 15x + C$$

$$= \frac{1}{4}x^4 - x^3 - \frac{5}{2}x^2 + 15x + C.$$

(5.) $I = \int (2x+3)^2 dx$

$= \int (4x^2+12x+9)\, dx$

$= 4\frac{x^3}{3} + 12\frac{x^2}{2} + 9x + C$

$= \frac{4}{3}x^3 + 6x^2 + 9x + C$

(6.) $I = \int e^{3x+4} dx$

$= \frac{e^{3x+4}}{3} + C$

(7) $I = \int \frac{1}{5x-1} dx$

$= \frac{\log|5x-1|}{5} + C.$

(8.) $I = \int 2^{3x-5} dx = \frac{2^{3x-5}}{(\log 2)\times 3} + C$

(9.) $I = \int (5x-2)^5 dx$

$= \frac{(5x-2)^6}{6 \times 5} + C$

$= \frac{(5x-2)^6}{30} + C$

(10.) $I = \int (5+e)\, dx$

$= 5\int dx + e\int dx$

$= 5x + ex + C.$

Practice Questions II

(1.) $TC = \int (3000\, e^{0.3x} + 50)\, dx = 3000 \int e^{0.3x} dx + 50 \int dx$

$TC = 3000 \times \frac{e^{0.3x}}{0.3} + 50x + K$

$TC = 10000\, e^{0.3x} + 50x + K.$

Fixed Cost = ₹80000 ⇒ ~~TC = 0 when~~

TC = 80000 when $x = 0$.

$80000 = 10000 \times e^{0.3\times 0} + 50\times 0 + K$

$80000 = 10000\times 1 + 0 + K$

$K = 70000$

$\therefore TC = 10000\, e^{0.3x} + 50x + 70000$

(2.) $MC = \cancel{6+10x*}\ 6+10x-6x^2$

$TC = \int MC\, dx = \int (6+10x-6x^2)\, dx$

$= 6x + \frac{\cancel{10}^{5} x^2}{\cancel{2}} - \frac{\cancel{6}^{2} x^3}{\cancel{3}} + K$

$TC = 6x + 5x^2 - 2x^3 + K$

when $x=1$ $\quad TC = 12$

$\therefore 12 = 6\times 1 + 5\times 1^2 - 2\times 1^3 + K$

$12 = 6+5-2+K$

$K = 3$

$TC = 6x + 5x^2 - 2x^3 + 3$

(3.) $MR = 20\, e^{-x/10}\left(1-\frac{x}{10}\right)$

$TR = \int MR\, dx = \int 20\, e^{-x/10}\left(1-\frac{x}{10}\right) dx$

$= 20 \int \underset{\text{II}}{e^{-x/10}} \underset{\text{I}}{\left(1-\frac{x}{10}\right)} dx$

$= 20\left[\left(1-\frac{x}{10}\right)\int e^{-x/10}\, dx - \int\left\{\left(1-\frac{x}{10}\right)'\cdot\int e^{-x/10}\, dx\right\} dx\right]$

(Applying Integration by parts)

$$= 20\left[\left(1-\frac{x}{10}\right)\cdot\frac{e^{-x/10}}{-1/10} - \int\left(-\frac{1}{10}\right)\cdot\frac{e^{-x/10}}{-1/10}\,dx\right].$$

$$= 20\left[-10\left(1-\frac{x}{10}\right)e^{-x/10} - \int -\frac{1}{\cancel{10}}\times -\cancel{10}\, e^{-x/10}\,dx\right]$$

$$= 20\left[-10\left(1-\frac{x}{10}\right)e^{-x/10} - \int e^{-x/10}\,dx\right]$$

$$= 20\left[-10\left(1-\frac{x}{10}\right)e^{-x/10} - \frac{e^{-x/10}}{-1/10}\right] + K$$

$$TR = 20\left[-10\left(1-\frac{x}{10}\right)e^{-x/10} + 10\,e^{-x/10}\right] + K.$$

if $x=0 \Rightarrow TR=0$

$$\therefore\ 0 = 20\left[-10\,(1-0)\,e^{0} + 10\,e^{0}\right] + K.$$

$$0 = 20\,(-10+10) + K.$$

$$K = 0.$$

$$\therefore\ TR = 20\left[-10\left(1-\frac{x}{10}\right)e^{-x/10} + 10\,e^{-x/10}\right].$$

$$= 200\,e^{-x/10}\left[-\cancel{1} + \frac{x}{10} + \cancel{1}\right].$$

$$= 20\cancel{0}\times\frac{x}{1\cancel{0}}\,e^{-x/10}$$

$$= 20x\,e^{-x/10}.$$

(4.) $TR = \int MR\, dx$

$$= \int (175 - 2x - 0.3x^2)\, dx$$

$$= 175x - \frac{2x^2}{2} - \frac{0.3x^3}{3} + C$$

$$TR = 175x - x^2 - 0.1x^3 + C \qquad C = 0 \quad \because TR = 0 \text{ when } x = 0$$

$$TR \text{ (when } x = 10) = 175 \times 10 - 10^2 - 0.1 \times 10^3$$
$$= 1750 - 100 - 100$$
$$= 1550$$

$$TR \text{ (when } x = 20) = 175 \times 20 - 20^2 - 0.1 \times 20^3$$
$$= 3500 - 400 - 800$$
$$= 2300.$$

$$\therefore \text{Inc. in } TR = 2300 - 1550$$
$$= ₹\ 750$$

Alternatively

$$TR = \int_{10}^{20} (175 - 2x - 0.3x^2)\, dx$$

$$= \left[175x - \frac{2x^2}{2} - 0.3\frac{x^3}{3}\right]_{10}^{20}$$

$$= \left[175x - x^2 - 0.1x^3\right]_{10}^{20}$$

$$= (175 \times 20 - 20^2 - 0.1 \times 20^3) - (175 \times 10 - 10^2 - 0.1 \times 10^3)$$

$$= 2300 - 1550$$

$$= ₹750.$$

5. $MR = 30 - 6x \qquad MC = -24 + 3x$

Marginal Profit = MP = MR − MC

$$= 30 - 6x - (-24 + 3x)$$
$$= 30 - 6x + 24 - 3x$$
$$= 54 - 9x$$

Total Profit (TP) $= \int (54 - 9x)\,dx$

$$TP = 54x - \frac{9x^2}{2} + K.$$

when $x = 0$ $\quad TP = -121.50$

$$\therefore \quad -121.50 = 54 \times 0 - \frac{9}{2} \times 0 + K$$
$$\therefore K = -121.50$$

$$TP = 54x - \frac{9}{2}x^2 - 121.50$$

For Break-even point put $TP = 0$

$$\text{i.e.} \quad 54x - \frac{9}{2}x^2 - 121.50 = 0$$
$$108x - 9x^2 - 243 = 0.$$
$$9x^2 - 108x + 243 = 0$$
$$x^2 - 12x + 27 = 0$$
$$x^2 - 9x - 3x + 27 = 0$$
$$(x-9)(x-3) = 0$$
$$x = 3, 9.$$

$\therefore$ Break-even points $\Rightarrow x = 3$ and $x = 9$.

Q6) $MC = 20 + \frac{x}{20}$ $MR = 30$.

Profit is maximum when

$MC = MR$

$20 + \frac{x}{20} = 30$.

$\frac{x}{20} = 10 \Rightarrow x = 200$

Marginal Profit $\Rightarrow$ $MR - MC$

$MP = 30 - \left(20 + \frac{x}{20}\right)$

$MP = 30 - 20 - \frac{x}{20}$

$= 10 - \frac{x}{20}$

Total Profit $TP = \int MP\,dx = \int \left(10 - \frac{x}{20}\right) dx$

$= 10x - \frac{1}{20} \cdot \frac{x^2}{2} + K$

$TP = 10x - \frac{x^2}{40} + K$.

when $x = 0$ TP (loss) $= -200$.

$\therefore -200 = 10 \times 0 - 0 + K$.

$\therefore TP = 10x - \frac{x^2}{40} - 200$.

To obtain max. profit put $x = 200$ (as calculated above)

$TP = 10 \times 200 - \frac{200^2}{40} - 200$

$= 2000 - \frac{\cancel{40000}^{1000}}{\cancel{40}} - 200$.

$= 2000 - 1200 =$ ₹800 (Max. Profit)

(7.) $MR = 9 - t^{1/3}$ $MC = 1 + 3t^{1/3}$

The operations should continue till we obtain the max. profits. i.e. when

$$MR = MC$$

$$9 - t^{1/3} = 1 + 3t^{1/3}$$

$$8 = 4t^{1/3}$$

$$2 = t^{1/3} \Rightarrow t = 2^3 \Rightarrow t = 8 \text{ yrs.}$$

Marginal Profit $\Rightarrow$ MR − MC

$$MP = 9 - t^{1/3} - 1 - 3t^{1/3}$$

$$MP = 8 - 4t^{1/3}$$

Total Profit $TP = \int (8 - 4t^{1/3})\,dt$

$$TP = 8t - 4 \cdot \frac{t^{4/3}}{4/3} + K.$$

$$TP = 8t - 3t^{4/3} + K$$

Since no information is given we assume

$TP = 0$ when $t = 0$.

$$\therefore 0 = 8 \times 0 - 3 \times 0 + K.$$

$$\therefore K = 0.$$

$$TP = 8t - 3t^{4/3}$$

For Max. profit put $t = 8$.

$$TP = 8 \times 8 - 3 \times 8^{4/3}.$$

$$= 64 - 3 \times (8^{1/3})^4$$

$$= 64 - 3 \times 2^4 = 64 - 48 = 16 \text{ million}$$

(8.) $\eta_d = \dfrac{3x}{2(3x+4)}$

$$-\frac{p}{x}\frac{dx}{dp} = \frac{3x}{2(3x+4)}$$

$$-\frac{2(3x+4)}{3x^2}\,dx = \frac{dp}{p}.$$

$$-\int \frac{6x+8}{3x^2}\,dx = \int \frac{dp}{p}$$

$$-\int\left(\frac{6x}{3x^2} + \frac{8}{3x^2}\right)dx = \int \frac{dp}{p}$$

$$-\int\left(\frac{2}{x} + \frac{8}{3}x^{-2}\right)dx = \int \frac{dp}{p}$$

$$-2\log x - \frac{8}{3}\cdot\frac{x^{-1}}{-1} + K = \log p.$$

$$\therefore \log p = -2\log x + \frac{8}{3x} + K.$$

when $p = 3$ $x = 5$

$$\therefore \log 3 = -2\log 5 + \frac{8}{3\times 5} + K.$$

$$\therefore K = \log 3 + 2\log 5 - \frac{8}{15}.$$

Hence Demand Function is:

$$\log p = -2\log x + \frac{8}{3x} + \log 3 + 2\log 5 - \frac{8}{15}$$

(9) $\frac{dC}{dI} = \frac{1}{\sqrt{I}}$

$dC = \frac{1}{\sqrt{I}} \, dI$

$\int dC = \int I^{-1/2} \, dI$

$C = \frac{I^{1/2}}{1/2} + K$

$C = 2\sqrt{I} + K.$

$C = 8$ when $I = 9$

$\therefore 8 = 2\sqrt{9} + K$

$8 = 2 \times 3 + K$

$K = 2$

$\therefore C = 2\sqrt{I} + 2$

(10) $\frac{dC}{dI} = 0.5 \Rightarrow \int dC = 0.5 \int dI$

$C = 0.5 I + K$

$C = 100$ when $I = 2$

$100 = 0.5 \times 2 + K \Rightarrow K = 99$

$\therefore \quad C = 0.5 I + 99$

Practice Questions III

(1) Demand: $p = 100 - 8x \qquad p = 4$

$\therefore \; 100 - 8x = 4$

$8x = 96 \Rightarrow x = 12$

when $p = 4 \quad x = 12$

Consumer's Surplus $CS = \int_0^{12} (100 - 8x)\, dx - 12 \times 4$

$= \left[100x - \frac{\cancel{8}^{4} x^2}{\cancel{2}} \right]_0^{12} - 48$

$= \left[(100 \times 12 - 4 \times 12^2) - (100 \times 0 - 4 \times 0) \right] - 48$

$= 1200 - 576 - 48 = 576.$

(2.) Supply: $p = \sqrt{9+x}$ $\qquad x = 7.$

when $x=7$ $\quad p = \sqrt{9+7} = \sqrt{16} = 4.$

$\therefore x = 7, \ p = 4.$

Producer's Surplus $PS = 7\times 4 - \int_0^7 \sqrt{9+x}\, dx.$

$$= 28 - \int_0^7 (9+x)^{1/2} dx$$

$$= 28 - \left|\frac{(9+x)^{3/2}}{3/2}\right|_0^7 = 28 - \frac{2}{3}\left|(9+x)^{3/2}\right|_0^7$$

$$= 28 - \frac{2}{3}\left((9+7)^{3/2} - (9+0)^{3/2}\right)$$

$$= 28 - 2/3\left(\left(16^{1/2}\right)^3 - \left(9^{1/2}\right)^3\right)$$

$$= 28 - 2/3\left(64 - 27\right)$$

$$= 28 - \frac{2}{3}\times 37$$

$$= \frac{84-74}{3} = \frac{10}{3}.$$

(3.) Demand: $p = \frac{8}{x+1} - 2$ $\qquad$ Supply: $p = \frac{1}{2}(x+3)$

At equilibrium Demand = Supply

$$\frac{8}{x+1} - 2 = \frac{1}{2}(x+3)$$

$$\frac{8-2(x+1)}{x+1} = \frac{x+3}{2}$$

$$\frac{8-2x-2}{x+1} = \frac{x+3}{2}$$

$$\frac{6-2x}{x+1} = \frac{x+3}{2}$$

$$12-4x = x^2+3x+x+3$$

$$12-4x = x^2+4x+3$$

$$\Rightarrow x^2+8x-9=0$$

$$x^2-9x+x-9=0$$

$$(x-9)(x+1)=0$$

$x = 9, -1$. Since x is qty. $\therefore x \geq 0$

Hence $x=9$

when $x=9$ $\quad p = \frac{1}{2}(9+3) = 6$.

Consumer's Surplus $CS = \int_0^9 \left(\frac{8}{x+1} - 2\right) dx - 9\times 6$.

$$= \left|8\log|x+1| - 2x\right|_0^9 - 54$$

$$= \left[(8\log 10 - 2\times 9) - (8\log 1 - 2\times 0)\right] - 54$$

$$= 8\log 10 - 18 - (0-0) - 54 \quad \because \log 1 = 0$$

$$= 8\log 10 - 72$$

Producer's Surplus $PS = 9\times 6 - \int_0^9 \frac{1}{2}(x+3)\,dx$

$$= 54 - \frac{1}{2}\left(\frac{x^2}{2} + 3x\right)_0^9$$

$$= 54 - \frac{1}{2}\left[\left(\frac{9^2}{2} + 3\times 9\right) - (0+0)\right].$$

$= 54 - \frac{1}{2}\left(\frac{81}{2} + 27\right)$

$= 54 - \frac{1}{2}\left(\frac{81+54}{2}\right)$

$= 54 - \frac{135}{4}$

$= \frac{216-135}{4} = \frac{81}{4}.$

(4.) Demand: $p = 50 - x^2$ $\qquad MC = 1 + x^2$

$TR = 50x - x^3$

$MR = 50 - 3x^2$

At. equilibrium $\Rightarrow$ $MR = MC$

$50 - 3x^2 = 1 + x^2$

$49 = 4x^2$

$x = \frac{7}{2}$

when $x = \frac{7}{2} \Rightarrow p = 50 - \left(\frac{7}{2}\right)^2$

$= 50 - \frac{49}{4}$

$= \frac{151}{4}.$

Consumer's Surplus $CS = \int_0^{7/2} (50 - x^2)\,dx - \frac{7}{2} \times \frac{151}{4}$

$= \left[50x - \frac{x^3}{3}\right]_0^{7/2} - \frac{1057}{8}.$

$= \left[50 \times \frac{7}{2} - \frac{1}{3} \times \left(\frac{7}{2}\right)^3\right] - \left[50 \times 0 - \frac{0}{3}\right] - \frac{1057}{8}$

$$= 175 - \frac{343}{24} - \frac{1057}{8}$$

$$= \frac{686}{24} = \frac{343}{12}$$

(5.) Demand: $p = 16 - x^2$ Supply: $p = 2x^2 + 4$

At equilibrium Demand = Supply

$$16 - x^2 = 2x^2 + 4$$

$$12 = 3x^2$$

$$x^2 = 4$$

$$x = 2$$

when $x = 2$ $p = 16 - 2^2 = 12$

Producer's Surplus $PS = 2 \times 12 - \int_0^2 (2x^2 + 4)\,dx$

$$= 24 - \left[2 \cdot \frac{x^3}{3} + 4x\right]_0^2$$

$$= 24 - \left[\left(\frac{2}{3} \cdot 2^3 + 4 \times 2\right) - \left(\frac{2}{3} \times 0 + 4 \times 0\right)\right]$$

$$= 24 - \left[\frac{16}{3} + 8\right]$$

$$= 24 - \frac{40}{3} = \frac{32}{3}.$$

Consumer's Surplus $CS = \int_0^2 (16 - x^2)\,dx - 2 \times 12$

$$= \left[16x - \frac{x^3}{3}\right]_0^2 - 24$$

$$= \left(16 \times 2 - \frac{8}{3}\right) - \left(16 \times 0 - \frac{0}{3}\right) - 24$$

$$= \frac{88}{3} - 24 = \frac{16}{3}$$

Exercise

(1.) Same as Practice Questions II Q5 (Solution at Pg 6)

(2.)

$$\frac{x}{C}\cdot\frac{dC}{dx} = \frac{9x}{2(9x+16)}$$

$$\int \frac{dC}{C} = \frac{9}{2}\int \frac{\cancel{x}}{\cancel{x}(9x+16)}\,dx$$

$$\int \frac{dC}{C} = \frac{9}{2}\int \frac{dx}{9x+16}$$

$$\log C = \frac{\cancel{9}}{2}\cdot\frac{\log(9x+16)}{\cancel{9}} + \log K.$$ $\because$ every constant can be written as log of some val.

$$\log C = \log(9x+16)^{1/2} + \log K$$ $\because a\log b = \log b^a$

$$\log C = \log\left[(9x+16)^{1/2}\times K\right]$$ $\because \log a + \log b = \log ab$

$$C = K\sqrt{9x+16}$$

Fixed Cost $= 16 \Rightarrow C = 16$ when $x = 0$.

$$16 = K\sqrt{9\times 0+16}$$

$$16 = K\sqrt{16} \Rightarrow 16 = 4K \Rightarrow K = 4.$$

$$\therefore C = 4\sqrt{9x+16}.$$

(3) Demand: $p = 30 - 4x - 4x^2$

when $p = 6 \Rightarrow \quad 30 - 4x - 4x^2 = 6.$

$$4x^2 + 4x - 24 = 0$$
$$x^2 + x - 6 = 0$$
$$x^2 + 3x - 2x - 6 = 0$$
$$(x+3)(x-2) = 0$$

$x = -3, 2 \qquad$ Since $x \geq 0$

$\therefore x = 2.$

Hence $x = 2$ when $p = 6$.

Consumer's Surplus $CS = \int_0^2 (30 - 4x - 4x^2)\,dx - 2 \times 6$

$$= \left[30x - \frac{\cancel{4}^{2} x^2}{\cancel{2}} - \frac{4x^3}{3}\right]_0^2 - 12$$

$$= \left[30 \times 2 - 2 \times 2^2 - \frac{4}{3} \times 2^3\right] - 12$$

$$= \left(60 - 8 - \frac{32}{3}\right) - 12$$

$$= 40 - \frac{32}{3}$$

$$= \frac{88}{3}.$$

4.
$$-\frac{\cancel{k}}{x} \times \frac{dx}{dp} = \frac{\cancel{k}}{x^{3}}$$

$$-\int x^2\,dx = \int dp$$

$$-\frac{x^3}{3} + K = P$$

$x=3,\ p=2 \Rightarrow \quad 2 = -\frac{3^3}{3} + K$

$2 = -9 + K \Rightarrow K = 11$

$$p = 11 - \frac{x^3}{3}.$$

5. Supply: $p = 4 + x \qquad x = 12 \Rightarrow p = 4 + 12 = 16$

Producer's Surplus $PS = \cancel{\int_0^{12}}\ 12\times 16 - \int_0^{12}(4+x)\,dx$

$$= 192 - \left(4x + \frac{x^2}{2}\right)_0^{12}$$

$$= 192 - \left[4\times 12 + \frac{12^2}{2}\right]$$

$$= 192 - (48 + 72)$$

$$= 72$$

Previous Years' Questions

(1.) Supply: $p = (x+2)^2$ $\quad p = 25$

$\therefore 25 = (x+2)^2$

$x+2 = \pm 5$

$\Rightarrow \quad x+2 = 5 \qquad x+2 = -5$

$x = 3 \qquad x = -7$

Since $x \geq 0 \Rightarrow x = 3$.

Producer's Surplus $PS = 25 \times 3 - \int_0^3 (x+2)^2 dx$

$= 75 - \left[\frac{(x+2)^3}{3}\right]_0^3$

$= 75 - \left[\frac{5^3}{3} - \frac{2^3}{3}\right]$

$= 75 - \frac{117}{3} = \frac{108}{3}$.

(2.) $MC = 0.003x^2 - 0.6x + 30$

$TC = \int MC\, dx = \int (0.003x^2 - 0.6x + 30)\, dx$

$TC = 0.003\frac{x^3}{3} - 0.6\frac{x^2}{2} + 30x + K$.

$TC = 0.001x^3 - 0.3x^2 + 30x + K$.

when $x = 0$ $\quad TC = 42$

$\therefore 42 = 0.001 \times 0 - 0.3 \times 0 + 30 \times 0 + K \Rightarrow K = 42$

Thus $TC = 0.001x^3 - 0.3x^2 + 30x + 42$

(3) $MR = 9 - 4x^2$

$TR = \int MR\,dx = \int (9-4x^2)\,dx$

$TR = 9x - 4\frac{x^3}{3} + K.$

$TR = 0$ when $x = 0 \Rightarrow 0 = 9\times 0 - \frac{4}{3}\times 0 + K \Rightarrow K = 0.$

$\therefore TR = 9x - \frac{4}{3}x^3$

Demand Function $= AR = \frac{TR}{x} = 9 - \frac{4}{3}x^2$

(4) $MC = 16x - 1591$ $AR = 9.$

~~TC = ∫(16x − 1519~~ $TR = 9x.$

$MR = 9$

Profit is max when $MC = MR$

$16x - 1591 = 9$

$16x = 1600$

$x = 100$

(5) See Practice Questions II Q7 (Pg 8)

(6) $P = 50 - x^2$ $MC = 1 + x^2.$

$TR = px = 50x - x^3$

$MR = 50 - 3x^2$

At equilibrium

$MR = MC$

$50 - 3x^2 = 1 + x^2$

$49 = 4x^2$

$x^2 = \frac{49}{4} \Rightarrow x = \frac{7}{2}.$

(7.) $MC = 4 - 2x + x^2$

$TC = \int MC\,dx$

$= \int (4 - 2x + x^2)\,dx = 4x - \frac{2x^2}{2} + \frac{x^3}{3} + K.$

$TC = 4x - x^2 + \frac{x^3}{3} + K.$

$TC = 100$ when $x = 0$.

$100 = 4\times0 - 0 + \frac{0}{3} + K \Rightarrow K = 100$

$\therefore TC = 4x - x^2 + \frac{x^3}{3} + 100.$

(8.) $MR = 2 - 6x$

$TR = \int MR\,dx = \int (2-6x)\,dx$

$TR = 2x - \frac{\cancel{6}^{3}x^2}{\cancel{2}} + K$

when $x = 0 \Rightarrow TR = 0$

$0 = 2\times0 - 3\times0 + K \Rightarrow K = 0$.

$\therefore TR = 2x - 3x^2$

Demand Function $= AR = \frac{TR}{x} = 2 - 3x.$

(9.) See Practice Questions II Q7 (Pg 8)

(10.) See Exercise Q4 (Pg 17)

(11.) ~~Demand: p = 100 - 8x~~ See Practice Questions III Q1 (Pg 10)

(12) See Exercise Q5 (Pg 17)

(13.) $MC = 2e^{0.001x}$

$TC = \int MC\,dx = 2\int e^{0.001x}\,dx$

$= 2 \cdot \dfrac{e^{0.001x}}{0.001} + K.$

$TC = 2000\,e^{0.001x} + K$

$TC = 2000$ when $x = 0$

$\therefore 2000 = 2000 \times e^{0.001 \times 0} + K$

$2000 = 2000 + K$

$K = 0.$

$\therefore TC = 2000\,e^{0.001x}$

(14.) Demand: $p = 18 - 2x - x^2$ Supply: $p = 2x - 3$

At Equilibrium Demand = Supply

$18 - 2x - x^2 = 2x - 3$

$x^2 + 4x - 21 = 0$

$x^2 + 7x - 3x - 21 = 0$

$(x+7)(x-3) = 0$

$x = -7, 3.$

Since $x \geq 0 \Rightarrow x = 3.$ Thus $p = 2 \times 3 - 3 = 3.$

Consumer's Surplus $CS = \int_0^3 (18 - 2x - x^2)\,dx - 3 \times 3.$

$= \left[18x - \cancel{2}\dfrac{x^2}{\cancel{2}} - \dfrac{x^3}{3}\right]_0^3 - 9$

$= \left[\left(18 \times 3 - 3^2 - \dfrac{3^3}{3}\right) - \left(18 \times 0 - 0 - \dfrac{0}{3}\right)\right] - 9$

$= 54 - 9 - 9 - 9$

$= 27.$

(15.) See Practice Questions III Q 5 (Pg 14)

(16.) See Practice Questions III Q1 (Pg 10)

(17.) $MR = 10 - 6x$

$TR = \int MR\,dx = \int (10-6x)\,dx$

$= 10x - \cancel{6}^{3}\frac{x^2}{\cancel{2}} + K$

$TR = 10x - 3x^2 + K$

$TR = 0$ when $x = 0$

$\therefore 0 = 10\times 0 - 3\times 0 + K$

$K = 0.$

$TR = 10x - 3x^2$

(18.) Demand: $p = 35 - 2x - x^2$ $x = 3.$

$\therefore p = 35 - 2\times 3 - 3^2$

$= 20.$

Consumer's Surplus $CS = \int_0^3 (35 - 2x - x^2)\,dx - 20\times 3.$

$= \left[35x - \cancel{2}\frac{x^2}{\cancel{2}} - \frac{x^3}{3}\right]_0^3 - 60.$

$= 35\times 3 - 9 - 9 - 60.$

$= 27.$

(19.) See Practice Questions III Q4 (Pg 13)

(20.) $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}.$

$\int dy = \frac{1}{2}\int \frac{1}{\sqrt{x}}\,dx.$

$\int dy = \frac{1}{2}\int x^{-1/2}\,dx.$

$y = \frac{1}{2}\cdot\frac{x^{1/2}}{1/2} + c.$

$y = \sqrt{x} + c.$

$y = 1$ $x = 1.$

$\therefore 1 = 1 + c \Rightarrow c = 0.$

$\therefore y = \sqrt{x}.$

21. $MC = 2(2x+9)^{-1/2}$

$$TC = 2\int (2x+9)^{-1/2}\,dx$$

$$TC = 2 \cdot \frac{(2x+9)^{1/2}}{\frac{1}{2}\times 2} + K.$$

$$TC = 2\sqrt{2x+9} + K.$$

Fixed Cost = 4

i.e. $TC = 4$ when $x = 0$.

$$4 = 2\sqrt{2\times 0+9} + K.$$

$$4 = 2\times\sqrt{9} + K.$$

$$4 = 6 + K.$$

$$K = -2.$$

$$\therefore\ TC = 2\sqrt{2x+9} - 2.$$

$$AC = \frac{2\sqrt{2x+9} - 2}{x}.$$

$$AC(x=8) = \frac{2\sqrt{2\times 8+9} - 2}{8} = \frac{2\times 5 - 2}{8} = \frac{8}{8} = 1.$$