04 DIFFERENTIATION & APPLICATIONS OF DIFFERENTIATION

Practice Questions I

(1) (i) $f(x) = 3x^2 - 7x + 6$ $\quad f'(x) = 6x - 7.$

(ii) $f(x) = 10x^{-3} + 5x^{-1/2} - 10x - 1$ $\quad f'(x) = -30x^{-4} - \frac{5}{2}x^{-3/2} - 10$

(iii) $f(x) = \dfrac{7x^3 + x}{2\sqrt{x}} = \dfrac{7}{2} \cdot \dfrac{x^3}{\sqrt{x}} + \dfrac{1}{2} \cdot \dfrac{x}{\sqrt{x}}$

$= \frac{7}{2} x^{5/2} + \frac{1}{2} \sqrt{x}$

$f'(x) = \frac{7}{2} \times \frac{5}{2} x^{3/2} + \frac{1}{2} \cdot \frac{1}{2\sqrt{x}} = \frac{35}{4} x^{3/2} + \frac{1}{4\sqrt{x}}$

(iv) $f(x) = \dfrac{5(x^4 - 7)}{8} = \dfrac{5}{8} x^4 - \dfrac{35}{8}$

$f'(x) = \frac{5}{\cancel{8}_2} \times \cancel{4}x^3 - 0 = \frac{5}{2} x^3$

(v) $f(x) = \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \times \cancel{x} \times \frac{1}{\cancel{x}} = x^2 + \frac{1}{x^2} + 2$

$= x^2 + x^{-2} + 2$

$f'(x) = 2x - 2x^{-3} = 2x - \frac{2}{x^3}.$

(2) (i) $f(x) = x^3 e^x$ $\quad f'(x) = (x^3)' e^x + x^3 (e^x)'$

$= 3x^2 e^x + x^3 e^x$

$= e^x (3x^2 + x^3).$

$= e^x x^2 (3 + x)$

(2) (ii) $f(x) = 3^x x^{3/2}$ $f'(x) = (3^x)' x^{3/2} + 3^x . (x^{3/2})'$

$= 3^x \log 3 . x^{3/2} + 3^x . \frac{3}{2} x^{1/2}$

$= 3^x . x^{3/2} \log 3 + \frac{1}{2} . 3^{x+1} x^{1/2}$

(iii) $f(x) = x \log x$ $f'(x) = x' \log x + x (\log x)'$

$= 1. \log x + x . \frac{1}{x}$

$= \log x + 1$

(iv) $f(x) = (5x^3 - 8) e^x$ $f'(x) = (5x^3-8)' e^x + (5x^3-8)(e^x)'$

$= 15x^2 e^x + (5x^3 - 8) e^x$

$= e^x (5x^3 + 15x^2 - 8)$

(v) $f(x) = x^2 \log x + 2^x$ $f'(x) = (x^2 \log x)' + (2^x)'$

$\therefore f'(x) = (x^2)' . \log x + x^2 (\log x)' + (2^x)'$

$= 2x \log x + x^2 \times \frac{1}{x} + 2^x \log 2$

$= 2x \log x + x + 2^x \log 2$

(vi) $f(x) = (16x^2 - x) e^x$

$f'(x) = (16x^2 - x)' e^x + (16x^2 - x)(e^x)'$

$= (32x - 1) e^x + (16x^2 - x) e^x$

$= e^x (32x - 1 + 16x^2 - x)$

$= e^x (16x^2 + 31x - 1)$

(2) (vii) $f(x) = x^5 e^x \log x$

$f'(x) = (x^5)' e^x \log x + x^5 (e^x)' \log x + x^5 e^x (\log x)'$

$= 5x^4 e^x \log x + x^5 e^x \log x + x^{\cancel{5}\,4} e^x \cdot \frac{1}{\cancel{x}}$

$= 5x^4 e^x \log x + x^5 e^x \log x + x^4 e^x$

~~$= x^4 e^x [5x \log x +$~~

$= x^4 e^x [5 \log x + x \log x + 1]$

(viii) $f(x) = 7^x (x^2+2x) e^x$

$f'(x) = (7^x)' (x^2+2x) e^x + 7^x (x^2+2x)' e^x + 7^x (x^2+2x)(e^x)'$

$= 7^x \log 7 (x^2+2x) e^x + 7^x (2x+2) e^x + 7^x (x^2+2x) e^x$

$= 7^x e^x [\log 7 (x^2+2x) + (2x+2) + (x^2+2x)]$

$= 7^x e^x [x^2 \log 7 + 2x \log 7 + 2x + 2 + x^2 + 2x]$

$= 7^x e^x [x^2 \log 7 + 2x \log 7 + x^2 + 4x + 2]$

(3) (i) $f(x) = \frac{2x+1}{x-1}$

$f'(x) = \frac{(x-1)(2x+1)' - (x-1)'(2x+1)}{(x-1)^2} = \frac{(x-1) \times 2 - 1(2x+1)}{(x-1)^2}$

$= \frac{\cancel{2x} - 2 - \cancel{2x} - 1}{(x-1)^2} = \frac{-3}{(x-1)^2}$

(3) (ii) $f(x) = \frac{x^3-x^2+1}{x^2-1}$

$$f'(x) = \frac{(x^2-1)(x^3-x^2+1)' - (x^2-1)'(x^3-x^2+1)}{(x^2-1)^2}$$

$$= \frac{(x^2-1)(3x^2-2x) - (2x)(x^3-x^2+1)}{(x^2-1)^2}$$

$$= \frac{3x^4 - \cancel{2x^3} - 3x^2 + \cancel{2x} - 2x^4 + \cancel{2x^3} - \cancel{2x}}{(x^2-1)^2}$$

$$= \frac{x^4-3x^2}{(x^2-1)^2}$$

(iii) $f(x) = \frac{\log x}{x^2}$

$$f'(x) = \frac{x^2(\log x)' - (x^2)'\log x}{(x^2)^2}$$

$$= \frac{\cancel{x^2} \times 1/\cancel{x} - 2x.\log x}{x^4}$$

$$= \frac{x - 2x\log x}{x^4} = \frac{\cancel{x}(1-2\log x)}{x^{\cancel{4}3}}$$

(iv) $f(x) = \frac{e^x}{\log x}$

$$f'(x) = \frac{\log x\,(e^x)' - (\log x)' e^x}{(\log x)^2}$$

$$= \frac{\log x\, e^x - \frac{1}{x}e^x}{(\log x)^2} = \frac{e^x\left(\log x - \frac{1}{x}\right)}{(\log x)^2}$$

$$= \frac{e^x(x\log x - 1)}{x(\log x)^2}$$

(v) $f(x) = \frac{5}{1-3x}$ $\quad f'(x) = \frac{(1-3x)\cdot(5)' - (1-3x)'(5)}{(1-3x)^2}$

$$= \frac{(1-3x)\times 0 - (-3)\times 5}{(1-3x)^2}$$

$$= \frac{15}{(1-3x)^2}$$

4. (i) $f(x) = (2x-5)^4$

$f'(x) = 4(2x-5)^3 \times (2x-5)'$

$= 4(2x-5)^3 \times 2 = 8(2x-5)^3$

(ii) $f(x) = \log x^3$

$f'(x) = \frac{1}{x^3} \times (x^3)' = \frac{1}{x^{\cancel{3}}} \times 3x^{\cancel{2}} = \frac{3}{x}.$

(iii) $f(x) = \log(\log x)$

$f'(x) = \frac{1}{\log x} \times (\log x)' = \frac{1}{\log x} \times \frac{1}{x} = \frac{1}{x\log x}.$

(iv) $f(x) = e^{2x-4}$

$f'(x) = e^{2x-4} \times (2x-4)' = e^{2x-4} \times 2 = 2e^{2x-4}.$

(4) (v) $f(x) = \frac{e^{2x}}{1+\log x}$

$$f'(x) = \frac{(1+\log x)(e^{2x})' - (1+\log x)'(e^{2x})}{(1+\log x)^2}$$

$$= \frac{(1+\log x)\, 2e^{2x} - \left(\frac{1}{x}\right) e^{2x}}{(1+\log x)^2}$$

$$= \frac{e^{2x}\left[2(1+\log x) - \frac{1}{x}\right]}{(1+\log x)^2}$$

$$= \frac{e^{2x}\left[2x(1+\log x) - 1\right]}{x(1+\log x)^2}$$

(vi) $f(x) = 4^{3x+1}$

$$f'(x) = 4^{3x+1} \log 4 \cdot (3x+1)'$$

$$= 4^{3x+1} \log 4 \times 3.$$

(vii) $f(x) = e^{x^2} \cdot \log x$

$$f'(x) = (e^{x^2})' \log x + e^{x^2} (\log x)'$$

$$= e^{x^2} \times 2x \log x + e^{x^2} \times \frac{1}{x}.$$

$$= e^{x^2}\left(2x\log x + \frac{1}{x}\right)$$

$$= e^{x^2}\left(\frac{2x^2\log x + 1}{x}\right)$$

(viii) $f(x) = \log(3x^2+2x-5)^3$

$$f'(x) = \frac{1}{(3x^2+2x-5)^3} \times \left[(3x^2+2x-5)^3\right]'$$

$$= \frac{1}{(3x^2+2x-5)^3} \times 3.(3x^2+2x-5)^2 \times (3x^2+2x-5)'$$

$$= \frac{3\,\cancel{(3x^2+2x-5)^2}}{(3x^2+2x-5)^{\cancel{3}}} \times (6x+2)$$

$$= \frac{3(6x+2)}{(3x^2+2x-5)^3}$$

(ix) $f(x) = \log(xe^x)$

$$f'(x) = \frac{1}{xe^x} \times (xe^x)'$$

$$= \frac{1}{xe^x} \times \left(x'e^x + x.(e^x)'\right)$$

$$= \frac{1}{xe^x}\left(e^x + xe^x\right) = \frac{\cancel{e^x}(1+x)}{x\cancel{e^x}} = \frac{1+x}{x}$$

(4) (x) $f(x) = (e^{3x}+1)^4$

$f'(x) = 4(e^{3x}+1)^3 \times (e^{3x}+1)'$

$= 4(e^{3x}+1)^3 \times e^{3x} \times (3x)'$

$= 4(e^{3x}+1)^3 \times e^{3x} \times 3$

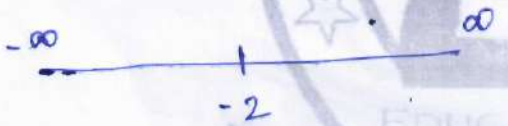
$= 12e^{3x}(e^{3x}+1)^3$

Practice Questions II

(1) (i) $f(x) = x^2+4x+3$ $\quad f'(x) = 2x+4$.

for Increasing/Decreasing Functions. $f'(x)=0$.

$\Rightarrow 2x+4=0 \Rightarrow x=-2$.



Interval	Assumed value	$f'(x)$	Result
$(-\infty,-2)$	$x=-3$	$-ve$	Decreasing Fn.
$(-2,\infty)$	$x=0$	$+ve$	Increasing Fn.

$\therefore$ The function is a Decreasing function when x lies in interval $(-\infty,-2)$

and it is ~~also~~ an Increasing function when x lies in the interval $(-2,\infty)$.

(ii) $f(x) = x^3-6x^2+9x$ $\quad f'(x) = 3x^2-12x+9$.

for Inc/Dec fn. $f'(x)=0 \Rightarrow 3x^2-12x+9=0$

$3x(x^2-4x+3)=0 \Rightarrow x^2-4x+3=0$

$\Rightarrow x^2-3x-x+3=0$

$\Rightarrow x(x-3)-1(x-3)=0$

$(x-3)(x-1)=0$

$x-3=0$ or $x-1=0$

$x=3$ $x=1$

$\therefore x=1,3$

$-\infty \quad 1 \quad 3 \quad \infty$

Interval	Assumed value	$f'(x)$	Result
$(-\infty, 1)$	$x=0$	$+ve$	Inc. Fn.
$(1,3)$	$x=2$	$-ve$	Dec fn.
$(3,\infty)$	$x=4$	$+ve$	Inc. Fn.

$\therefore f(x)$ is an Inc. Fn. when x lies in the interval ~~$(-\infty,1)\cup(3,\infty)$~~ $[(-\infty,1)\cup(3,\infty)]$

$f(x)$ is a dec. fn. when x lies in the interval $(1,3)$.

(2.) $R(x) = 160000 - (x-400)^2$

$R'(x) = -2(x-400)$

For Inc/Dec fn. $R'(x)=0 \Rightarrow -2(x-400)=0$.

$\Rightarrow x-400=0$ or $x=400$.

$-\infty \quad 400 \quad \infty$

Interval	Assumed val.	$R'(x)$	Result
$(-\infty, 400)$	$x=0$.	$+$	Increasing Fn.
$(400,\infty)$	$x=500$.	$-$	Dec. Fn.

(3.) $AC = x + 5 + \frac{36}{x}$

$AC' = 1 + 0 - \frac{36}{x^2} = 1 - \frac{36}{x^2}$

For Inc. or Dec fn. $AC' = 0 \Rightarrow 1 - \frac{36}{x^2} = 0$

$1 = \frac{36}{x^2} \Rightarrow x^2 = 36.$

$x = 6.$

$-\infty \quad 6 \quad \infty.$

Interval	Assumed val	AC'	Result
$(-\infty, 6)$	$x = 1$	$-$	Dec fn.
$(6, \infty)$	$x = 7$	$+$	Inc Fn.

(4.) $f(x) = x^4 - \frac{x^3}{3}.$

$f'(x) = 4x^3 - \frac{\cancel{3}x^2}{\cancel{3}} = 4x^3 - x^2 = x^2(4x - 1)$

for Inc./Dec fn. $f'(x) = 0$ $x^2(4x-1) = 0$.

Since $x^2 > 0$ for all x $\therefore$ x^2 is ignored and value of x is found by putting $4x - 1 = 0$

$\therefore x = \frac{1}{4}.$

$-\infty \quad \frac{1}{4} \quad \infty$

Interval	Assumed val	$f'(x)$	Result
$(-\infty, \frac{1}{4})$	$x = -1$	$-$	Dec. Fn.
$(\frac{1}{4}, \infty)$	$x = 1$	$+$	Inc Fn.

(5.) $p = 5x + 3$

$p' = 5$

Since $p' > 0$ for all x $\therefore$ given Supply fn. is an Increasing fn. for all x.

Practice Question III

(1.) $f(x) = x^3 - 3x^2 + 3x + 3$

$f'(x) = 3x^2 - 6x + 3$ $\quad f''(x) = 6x - 6$.

for Concavity / Convexity put $f'(x) = 0$. [We can also put $f''(x) = 0$ & find x].

$3x^2 - 6x + 3 = 0$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x - 1 = 0 \Rightarrow x = 1$.

Interval	Assumed val.	$f''(x)$	Result
$(-\infty, 1)$	$x = 0$	$-$	Concave Down
$(1, \infty)$	$x = 2$	$+$	Concave Up
$x = 1$		0	Pt. of Inflexion.

(2.) $C(x) = 0.001x^3 - 0.3x^2 + 30x + 42$

$C'(x) = 0.003x^2 - 0.6x + 30$.

$= \frac{3}{1000}x^2 - \frac{6}{10}x + 30 \Rightarrow C'(x) = \frac{3x^2 - 600x + 30000}{1000}$

For Concavity | Convexity

Put $C'(x) = 0$ $\quad \frac{3x^2 - 600x + 30000}{1000} = 0$.

$3x^2 - 600x + 30000 = 0$.

$x^2 - 200x + 10000 = 0$.

$(x-100)^2 = 0 \Rightarrow x = 100$

Interval	Assumed val.	$C''(x)$	Result
$(-\infty, 100)$	$x = 0$	$-$	Concave Down
$(100, \infty)$	$x = 200$	$+$	Concave Up
$x = 100$		0	Pt of Inflexion

$-\infty$ ——— 100 ——— ∞

$C''(x) = 0.006x - 0.6$.

(3.) $y = x^2 + 4$

$\frac{dy}{dx} = 2x \Rightarrow \frac{d^2y}{dx^2} = 2$

Since Second Derivative $\left(\frac{d^2y}{dx^2}\right)$ is positive for all x.

$y = x^2 + 4$ is a Concave Up function.

(4.) $y = 2x - 3 + \frac{1}{x}$.

$\frac{dy}{dx} = 2 - 0 - \frac{1}{x^2} \Rightarrow \frac{dy}{dx} = 2 - \frac{1}{x^2} = 2 - x^{-2}$

$\frac{d^2y}{dx^2} = 2x^{-3} = \frac{2}{x^3}$.

for Concavity | Convexity $\frac{dy}{dx} = 0$.

Wrong Question.

$$2 - \frac{1}{x^2} = 0$$

$$2 = \frac{1}{x^2}$$

$$x^2 = \frac{1}{2} \Rightarrow x = \frac{1}{\sqrt{2}} = 0.7$$

$-\infty$ ——|—— ∞.

0.7

Interval	Assumed val.	$\frac{d^2y}{dx^2}$	Result
$(-\infty, 0.7)$	1	+	Concave Up

(5.) $p = \frac{50}{x+3}$

$$p' = \frac{-50}{(x+3)^2}$$

$$p'' = -50 \times \left[(x+3)^{-2}\right]'$$

$$= -50 \times \left(-2(x+3)^{-3}\right)$$

$$= \frac{100}{(x+3)^3}$$

Since $p' < 0$ for all x $\therefore$ $p = \frac{50}{x+3}$ is a Decreasing Fn.

In demand function x represents quantity.

$\therefore x \geq 0$.

Hence $p'' = \frac{100}{(x+3)^3} > 0$ for all x

$\therefore$ $p = \frac{50}{x+2}$ is a Concave Up.

Practice Question IV

1.
$$f(x) = 2x^3 - 15x^2 + 36x + 18$$
$$f'(x) = 6x^2 - 30x + 36$$
For Maxima or Minima
$$f'(x) = 0 \Rightarrow 6x^2 - 30x + 36 = 0$$
$$x^2 - 5x + 6 = 0$$
$$(x-3)(x-2) = 0$$
$$x = 3, 2.$$

$$f''(x) = 12x - 30$$
$$f''(x=3) = 12 \times 3 - 30 = 6 > 0$$
$\therefore x = 3$ is the pt. of ~~Maxima~~ Minima
$$f''(x=2) = 12 \times 2 - 30 = -6 < 0$$
$x = 2$ is the pt. of Maxima.

2. (i) $f(x) = 3x^3 - 4x^2 + 1 \Rightarrow f'(x) = 9x^2 - 8x$
For Maxima or Minima $f'(x) = 0$
$$\Rightarrow 9x^2 - 8x = 0 \quad x(9x - 8) = 0$$
$$x = 0 \quad 9x - 8 = 0$$
$$x = 8/9.$$

$$f''(x) = 18x - 8$$
$$f''(x=0) = 18 \times 0 - 8 = -8 < 0.$$
Maxima.
$\therefore x = 0$ is the pt. of Maxima.
$$f''(x = 8/9) = 18 \times \frac{8}{9} - 8 = 8 > 0$$
$\therefore x = 8/9$ is the pt of Minima.

(3) (iii) $f(x) = x^3 - 6x^2 + 9x - 8$

$f'(x) = 3x^2 - 12x + 9$

For Maxima/Minima $f'(x) = 0$

$3x^2 - 12x + 9 = 0 \Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow x^2 - 3x - x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3, 1.$

$f''(x) = 6x - 12$

$f''(x=3) = 6 \times 3 - 12 = 6 > 0$

$\therefore x = 3$ is the pt of Minima.

$f''(x=1) = 6 \times 1 - 12 = -6 < 0$

$\therefore x = 1$ is the pt of Maxima.

(2) (ii) $f(x) = 2x^3 - 3x^2 - 4x + 4$

$f'(x) = 6x^2 - 6x - 4$

For Maxima or Minima $f'(x) = 0$

$\Rightarrow 6x^2 - 6x - 4 = 0 \Rightarrow 3x^2 - 3x - 2 = 0.$

Factors not possible.

(4.) $3y = x^3 - 3x^2 - 9x + 11$

$f(x) = y = \frac{1}{3}x^3 - x^2 - 3x + \frac{11}{3}.$

$f'(x) = x^2 - 2x - 3.$

For Maxima/Minima $\Rightarrow f'(x) = 0.$

$x^2 - 2x - 3 = 0 \Rightarrow x^2 - 3x + x - 3 = 0.$

$(x-3)(x+1) = 0.$

$x = 3, -1.$

4. Continued

$f''(x) = 2x - 2$

$f''(x=3) = 2 \times 3 - 2 = 4 > 0$

$x=3$ is the pt of Minima.

$f''(x=-1) = 2 \times (-1) - 2 = -4 < 0$

$x=-1$ is the pt. of Maxima.

$f''(x=1) = 2 \times 1 - 2 = 0$.

$\therefore x=1$ is the pt. of inflexion.

5. $f(x) = 2x^3 - 15x^2 + 36x + 18$

$f'(x) = 6x^2 - 30x + 36$.

For Maxima/Minima $\Rightarrow f'(x) = 0$

$6x^2 - 30x + 36 = 0$

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$x = 3, 2$.

$\boxed{\text{Same as Q1}}$

Practice Questions V

① $C = 0.1x^2 + 3x + 1$

(i) Average Cost $= AC = \frac{C}{x} = \frac{0.1x^2+3x+1}{x}$

$= 0.1x + 3 + \frac{1}{x}$.

(ii) Marginal Cost $= MC = C' = 0.2x + 3$.

(iii) $MC(x=5) = 0.2 \times 5 + 3 = 4$.

② $C(x) = 0.005x^3 - 0.02x^2 - 30x + 5000$

(i) Average Cost $= AC = \frac{C(x)}{x} = \frac{0.005x^3 - 0.02x^2 - 30x + 5000}{x}$

$= 0.005x^2 - 0.02x - 30 + \frac{5000}{x}$.

(ii) Marginal Cost $= MC = C'(x) = 0.015x^2 - 0.04x - 30$.

(iii) Marginal Average Cost $= MAC = AC'(x)$

$= 0.01x - 0.02 - \frac{5000}{x^2}$

(iv) Rate of change of Marginal Cost

$= MC'(x) = 0.03x - 0.04$

③ Under pure competition

$p = ₹15$

$\therefore TR = px = 15x$.

$AR = \frac{TR}{x} = \frac{15x}{x} = 15$

$MR = (TR') = 15$.

(4.) $x = 10 - 2p$.

$2p = 10 - x$

$p = 5 - \frac{x}{2}$

$TR = px = \left(5 - \frac{x}{2}\right)x = 5x - \frac{x^2}{2}$

$AR = \frac{TR}{x} = \frac{5x - \frac{x^2}{2}}{x} = 5 - \frac{x}{2}$.

$MR = TR' = 5 - x$.

(5.) $R(x) = 20x - \frac{x^2}{2}$

(i) $AR = \frac{R(x)}{x} = \frac{20x - x^2/2}{x} = 20 - \frac{x}{2}$

(ii) $MR = R'(x) = 20 - \frac{1}{\cancel{2}} \times \cancel{2}x = 20 - x$.

(iii) $MR(x=5) = 20 - 5 = 15$

Practice Questions VI

1. Demand Fn $\Rightarrow p = -0.5x + 20$

Total Revenue ~~Product~~ $= px = (-0.5x + 20)x$

$= -0.5x^2 + 20x$.

Total Revenue Product $= TRP = -0.5(2t)^2 + 20(2t)$

$= -0.5 \times 4t^2 + 20 \times 2t$

$= -2t^2 + 40t$.

Marginal Revenue Product

$(MRP) = -4t + 40$

$MRP(t=5) = \cancel{-4 \times 40 + 40} = -4 \times 5 + 40$

$\cancel{= -160 + 40} \qquad = -20 + 40$

$\cancel{= 120} \qquad = 20$

(2.) $p = -0.1x + 70$

$TR = px = -0.1x^2 + 70x$

$TRP = -0.1\left(\frac{200t - t^2}{20}\right)^2 + 70\left(\frac{200t - t^2}{20}\right)$

$= -0.1\frac{(200t - t^2)^2}{400} + \frac{70}{20}(200t - t^2)$

$= -0.00025(200t - t^2)^2 + 3.5(200t - t^2)$

$MRP = -0.00025 \times 2(200t - t^2)(200 - 2t) + 3.5(200 - 2t)$

$= (200 - 2t)\left[-0.0005(200t - t^2) + 3.5\right]$

$= (200 - 2t)(-0.1t + 0.0005t^2 + 3.5)$

$MRP(t = 40) = (200 - 2 \times 40)(-0.1 \times 40 + 0.0005 \times 40^2 + 3.5)$

$= 120 \times (-4 + 0.8 + 3.5)$

$= 120 \times 0.3$

$= 36$.

Practice Questions VII

(1.) $$C = 6 + \frac{3}{4} I - \frac{1}{3}\sqrt{I}$$

$$MPC = C' = \frac{3}{4} - \frac{1}{3} \times \frac{1}{2\sqrt{I}}$$

$$C' = \frac{3}{4} - \frac{1}{6\sqrt{I}}$$

$$MPC\,(I = 25) = \frac{3}{4} - \frac{1}{6 \times \sqrt{25}}$$

$$= \frac{3}{4} - \frac{1}{6 \times 5} =$$

$$= \frac{3}{4} - \frac{1}{30} = \frac{90 - 4}{120} = \frac{86}{120} = 0.716$$

(2.) $$C = 2 + 2\sqrt{I}$$

$$MPC = C' = \cancel{2} \times \frac{1}{\cancel{2}\sqrt{I}} = \frac{1}{\sqrt{I}}$$

$$MPC\,(I = 100) = \frac{1}{\sqrt{100}} = \frac{1}{10} = \underline{0.1}$$

Practice Questions VIII

(1.) $$x = 25 - 3p - p^2 \Rightarrow \frac{dx}{dp} = \cancel{-3p - 2p}\ -3 - 2p$$

$$\frac{dx}{dp} = -3 - 2 \times 5 = -13.$$

$p = 5$ $$x = 25 - 3 \times 5 - 5^2$$

$$= \cancel{25} - 15 - \cancel{25} = -15.$$

$$\eta_d = -\frac{p}{x} \cdot \frac{dx}{dp} = +\frac{5}{+15} \times (-13) = (-)\,4.33.$$

(2.) $p = 4 - 5x^2$

$\frac{dp}{dx} = -10x \Rightarrow \frac{dx}{dp} = \frac{-1}{10x}$

$\eta_d = -\frac{p}{x} \times \frac{dx}{dp}$

$= -\frac{(4-5x^2)}{x} \times \frac{-1}{10x}.$

$= \frac{4-5x^2}{10x^2}$

Demand is unit elastic

$\therefore \eta_d = 1.$

$\frac{4-5x^2}{10x^2} = 1$

$4 - 5x^2 = 10x^2$

$4 = 15x^2$

$x^2 = \frac{4}{15}$

$x = \frac{2}{\sqrt{15}}.$

(3.) $q = 500 - 2p.$

$\frac{dq}{dp} = -2.$

$\eta_d = -\frac{p}{q} \cdot \frac{dq}{dp}$

$= -\frac{p}{500-2p} \times (-2)$

$= \frac{2p}{500-2p}$

For elastic demand $\eta_d > 1$

$\frac{2p}{500-2p} > 1$

$2p > 500 - 2p.$

$4p > 500$

$p > \frac{500}{4} \Rightarrow p > 125$

For Unit elastic demand $\eta_d = 1.$

$\frac{2p}{500-2p} = 1 \Rightarrow p = 125$

For Inelastic demand $\eta_d < 1$

$\frac{2p}{500-2p} < 1 \Rightarrow p < 125.$

(4.) $x = 5 + \frac{1}{10} I$

$\frac{dx}{dI} = \frac{1}{10}.$

when $I = 100$ $x = 5 + \frac{1}{\cancel{10}} \times 10\cancel{0}$

$= 15$

$\therefore \eta_I = \frac{I}{x} \cdot \frac{dx}{dI}$

$= \frac{10\cancel{0}}{15} \times \frac{1}{1\cancel{0}}$

$= 0.66.$

(5.) $x = -75 + \frac{1}{2} I \Rightarrow \frac{dx}{dI} = \frac{1}{2}.$

when $I = 400$ $x = -75 + \frac{1}{2} \times \cancel{400}\ 200$

$= 125$

$\eta_I = \frac{I}{x} \times \frac{dx}{dI} = \frac{\overset{200}{\cancel{400}}}{125} \times \frac{1}{\cancel{2}} = \frac{8}{5} = 1.6.$

(6.) $MR = AR\left(\frac{e-1}{e}\right)$

$MR = 20\left(\frac{2-1}{2}\right)$

$= \overset{10}{\cancel{20}} \times \frac{1}{\cancel{2}} = 10.$

(7) $MR = AR\left(\frac{e-1}{e}\right)$

$10 = 35\left(\frac{e-1}{e}\right).$

or $e = \frac{AR}{AR - MR}$

$= \frac{35}{35-10} = \frac{35}{25} = 1.4$

(8) $MR = AR\left(\frac{e-1}{e}\right)$

$-15 = AR\left(\frac{0.75-1}{0.75}\right) \Rightarrow +11.25 = AR \times (+0.75)$

$\underline{AR = 45}$

(9.) $p = \sqrt{9+x}$. when $p = 7$

$7 = \sqrt{9+x} \Rightarrow 49 = 9 + x$

$\Rightarrow x = 40$.

$\frac{dp}{dx} = \frac{1}{2\sqrt{9+x}}$

$\frac{dp}{dx} = \frac{1}{2\sqrt{9+40}} = \frac{1}{2\times 7} = \frac{1}{14}. \Rightarrow \frac{dx}{dp} = 14.$

$\eta_s = \frac{p}{x} \cdot \frac{dx}{dp}$

$= \frac{7}{40} \times 14$

$= 2.45$

(10.) Let the supply function be $x = a + bp$

Case I when $p = ₹350 \Rightarrow x = 50$.

$50 = a + b\times 350 \Rightarrow 50 = a + 350b$ — (1)

Case II when $p = ₹300 \Rightarrow x = 35$.

$35 = a + b\times 300 \Rightarrow 35 = a + 300b$ — (2).

Solving (1) & (2).

$$\begin{array}{r} a + 350b = 50 \\ a + 300b = 35 \\ (-) \quad (-) \quad (-) \\ \hline 50b = 15 \end{array}$$

$\therefore b = \frac{15}{50}$.

(1) $\Rightarrow 50 = a + \cancel{350}^{7} \times \frac{15}{\cancel{50}}$

$a = 50 - 105$

$= -55$

$\therefore$ Supply Function $\Rightarrow$

$$\cancel{p} \; x = -55 + \frac{15}{50} p$$

$$\frac{dx}{dp} = \frac{15}{50}$$

$$x = 50 \qquad p = 350.$$

$$\eta_s = \frac{p}{x} \times \frac{dx}{dp}$$

$$= \frac{\cancel{350}^{7}}{\cancel{50}} \times \frac{\cancel{15}^{3}}{\cancel{50}_{10}}.$$

$$= \underline{\underline{2.1}}.$$

(11.)

$$C = \frac{1}{6}x^3 + 642x + 400$$

$$\frac{dC}{dx} = \frac{1}{\cancel{6}_2} \times \cancel{3}x^2 + 642 = \frac{1}{2}x^2 + 642$$

$$\eta_c = \frac{x}{C} \times \frac{dC}{dx}.$$

$$= \frac{x}{\frac{1}{6}x^3 + 642x + 400} \times \frac{1}{2}x^2 + 642.$$

$$= \frac{\frac{1}{2}x^3 + 642x}{\frac{1}{6}x^3 + 642x + 400}.$$

(12.) $C = 2x^2+4x+3$

$\frac{dC}{dx} = 4x+4$

$\eta_C = \frac{x}{C} \times \frac{dC}{dx}$

$= \frac{x}{2x^2+4x+3} \times (4x+4)$

$= \frac{4x^2+4x}{2x^2+4x+3}$

$AC = 2x+4+3/x.$

$\frac{d}{dx}(AC) = 2 - \frac{3}{x^2}.$

$\eta_{AC} = \frac{x}{AC} \times \frac{d.(AC)}{dx}$

$= \frac{x}{2x+4+\frac{3}{x}} \times 2 - 3/x^2$

$= \frac{2x - \frac{3}{x}}{2x+4+\frac{3}{x}}.$

Practice Questions IX

(1.) $D = 10000 \quad C_0 = 50 \quad C_H = 1.$

$\therefore C = \frac{D}{Q} C_0 + \frac{Q}{2} . C_H$

$C = \frac{10000}{Q} \times 50 + \frac{Q}{2} \times 1$

$C = \frac{500000}{Q} + \frac{Q}{2}.$

$C' = -\frac{500000}{Q^2} + \frac{1}{2}.$

For Max. or Min $\Rightarrow$ $C' = 0.$

$-\frac{500000}{Q^2} + \frac{1}{2} = 0 \Rightarrow 1000000 = Q^2$

$Q = 1000.$

C''

$C' = -500000Q^{-2} + \frac{1}{2}$

$C'' = 1000000Q^{-3}$

$C'' = \frac{1000000}{Q^3} > 0$

$\therefore C$ is Min. when $Q = 1000$.

(2.) $D = 20 \times 250$
$= 5000$

$C_0 = ₹40$

$C_H = 2\%$ of $₹80$

$= \frac{2}{100} \times 80 = ₹1.6$.

$\therefore C = \frac{D}{Q} \times C_0 + \frac{Q}{2} \times C_H$

$C = \frac{5000}{Q} \times 40 + \frac{Q}{2} \times \overset{0.8}{\cancel{1.6}}$

$C = \frac{200000}{Q} + 0.8Q$.

$C' = -\frac{200000}{Q^2} + 0.8$

For Max / Min $\Rightarrow C' = 0$.

$-\frac{200000}{Q^2} + 0.8 = 0$.

$Q^2 = 250000$

$Q = 500$.

$C' = -200000Q^{-2} + 0.8$

$C'' = 200000Q^{-3}$

$C'' = \frac{200000}{Q^3} > 0$

$\therefore C$ is Min when $Q = 500$.

Applied Max-Min Problems

Practice Questions X

(1.) $y = e^{-x/3}$

$TR = yx = e^{-x/3} \cdot x.$

$TR' = (e^{-x/3})' \cdot x + e^{-x/3} \cdot (x)'$

$= e^{-x/3} \times \left(-\frac{1}{3}\right) x + e^{-x/3} \times 1$

$= e^{-x/3} \left[-\frac{1}{3}x + 1 \right]$

For Maxima / Minima $TR' = 0$.

$e^{-x/3}\left(-\frac{1}{3}x + 1\right) = 0$

$-\frac{1}{3}x + 1 = 0$

$x = 3.$

$TR'' = (e^{-x/3})' \left(1 - \frac{1}{3}x\right) + e^{-x/3}\left(1 - \frac{1}{3}x\right)'$

$= e^{-x/3}\left(-\frac{1}{3}\right)\left(1 - \frac{1}{3}x\right) + e^{-x/3}\left(-\frac{1}{3}\right).$

$= -\frac{1}{3} e^{-x/3} \left[1 - \frac{1}{3}x + 1 \right]$

$= -\frac{1}{3} e^{-x/3} \left[1 - \frac{1}{\cancel{3}} \times \cancel{3} + 1 \right]$

$= -\frac{1}{3} e^{-x/3} < 0$ $\quad \therefore$ TR is Max when $x = 3$

$y = e^{-3/3} = e^{-1}$

Pg

(2.) $TC = \frac{1}{3}x^3 - 18x^2 + 160x$

$AC = \frac{1}{3}x^2 - 18x + 160.$

$AC' = \frac{2}{3}x - 18$

For Max / Min

$AC' = 0 \quad x = 18 \times \frac{3}{2} = 27$

$AC'' = \frac{2}{3} > 0$

AC is Min when $x = 27$

$MC = x^2 - 36x + 160.$

$MC' = 2x - 36.$

For Max / Min

$MC' = 0 \quad x = 18.$

$MC'' = 2 > 0$

MC is Min when $x = 18$.

(3.) $C = 100 + 0.015x^2 \quad R = 3x.$

Profit $P = R - C$

$= 3x - (100 + 0.015x^2)$

$= 3x - 100 - 0.015x^2$

$P = -0.015x^2 + 3x - 100$

$P' = -0.03x + 3.$

For Max / Min

$P' = 0 \quad x = \frac{3}{0.03} = 100.$

$P'' = -0.03 < 0$

$\therefore$ Profit is Maximised when $x = 100$.

(4.) $p = \frac{50}{\sqrt{x}}$

$TR = px = \frac{50}{\sqrt{x}} \times x$

$= 50\sqrt{x}.$

$AC = 0.5 + \frac{1000}{x}$

$TC = x \cdot AC$

$= 0.5x + 1000.$

$Profit = TR - TC$

$P = 50\sqrt{x} - 0.5x - 1000$

$P' = \cancel{50}^{25} \times \frac{1}{\cancel{2}\sqrt{x}} - 0.5$

For Max / Min $\Rightarrow P' = 0$

$\frac{25}{\sqrt{x}} - 0.5 = 0.$

$\frac{25}{0.5} = \sqrt{x}.$

$50 = \sqrt{x}$

$x = 2500.$

$P' = 25x^{-1/2} - 0.5$

$P'' = -\frac{25}{2}x^{-3/2} < 0$

$\therefore P$ is Max when

$x = 2500.$

$MR = \frac{\cancel{50}^{25}}{\cancel{2}\sqrt{x}}.$

$= \frac{25}{\sqrt{2500}} = \frac{25}{50} = \frac{1}{2}$

$MC = 0.5 = \frac{1}{2}.$

$\therefore$ when Profit is Maximum

$MR = MC.$

(5.) Average repair Cost = ₹50 No. of repairs = $t^{3/2}$

$\therefore$ Total Cost of repair = $50t^{3/2}$

Replacement Cost = ₹20000.

$\therefore$ Total Cost = $20000 + 50t^{3/2}$

Optimal replacement Period will be when AC is minimum.

$$AC = \frac{\text{Total Cost}}{t} = \frac{20000 + 50t^{3/2}}{t}.$$

$$= \frac{20000}{t} + 50t^{1/2}$$

$$AC' = -\frac{20000}{t^2} + \frac{\cancel{50}^{25}}{\cancel{2}\sqrt{t}}.$$

$$AC' = 0 \Rightarrow -\frac{20000}{t^2} + \frac{25}{t^{1/2}} = 0.$$

$$\frac{+20000}{t^2} = \frac{25}{t^{1/2}}.$$

$$\frac{\cancel{20000}^{800}}{\cancel{25}} = \frac{t^2}{t^{1/2}}$$

$$800 = t^{3/2}$$

$$t = (800)^{2/3}.$$

(6.) Cost of Machinery = ₹12000

Operation Cost = $20t^2+15t$

Scrap Val. = $6880-60t^2$

$\therefore$ Total Cost = $12000+20t^2+15t-(6880-60t^2)$

$TC = 12000+20t^2+15t-6880+60t^2$

$TC = 80t^2+15t+5120.$

$\Rightarrow AC = 80t+15+\dfrac{5120}{t}$

$AC' = 80-\dfrac{5120}{t^2}$

For Maxima / Minima $\quad AC'=0$

$80-\dfrac{5120}{t^2}=0 \Rightarrow \quad t^2=\dfrac{5120}{80}=64$

$t=8.$

$\therefore$ Optimum time for replacement = 8 yrs.

(7.) $p=400-2x \Rightarrow TR=400x-2x^2$

$AC=0.2x+4+\dfrac{4}{x} \Rightarrow TC=0.2x^2+4x+4.$

<u>Before Imposition of Tax</u>

Profit = $TR-TC = 400x-2x^2-0.2x^2-4x-4$

$P = -2.2x^2+396x-4$

$P' = -4.4x+396.$

For Max / Min $\Rightarrow P'=0 \Rightarrow x=90.$

After Imposition of tax

$TR = 400x - 2x^2$ $\qquad TC = 0.2x^2 + 4x + 4 + 22x.$

$\qquad\qquad\qquad\qquad\qquad = 0.2x^2 + 26x + 4$

$\text{Profit} = P = 400x - 2x^2 - 0.2x^2 - 26x - 4$

$= -2.2x^2 + 374x - 4.$

$P' = -4.4x + 374$

For Max/Min $\Rightarrow P' = 0$

$-4.4x + 374 = 0$

$x = 85.$

(8) Let the no. of vacant apartments be x.

$\therefore$ Total no. of apartments on rent $= 60 - x$.

Monthly Rent $= 450 + 15x$.

$TR = (60-x)(450 + 15x)$

$= 27000 + 900x - 450x - 15x^2$

$= -15x^2 + 450x + 27000.$

$TC = 60(60-x) = 3600 - 60x.$

$\therefore$ Profit $P = TR - TC$

$= -15x^2 + 450x + 27000 - 3600 + 60x$

$= -15x^2 + 510x + 23400$

$P' = -30x + 510$

For Maxima / Minima $\Rightarrow P' = 0$

$-30x + 510 = 0 \Rightarrow x = 17.$

∴ Profit is max when no. of vacant apts. = 17.

(9.) Let the no. of units in excess of 50 be x.

∴ Order Size = $50+x$

Charge per set = $550-5x$.

$TR = (50+x)(550-5x)$

$= 27500 - 250x + 550x - 5x^2$

$= -5x^2 + 300x + 27500$

$TR' = -10x + 300$

For Max./Min ⇒ $TR' = 0$

$-10x + 300 = 0$

$x = 30$.

Revenue is Maximised when $x = 30$.

(10.) Let the increase in monthly subscription be 'x'.

∴ Monthly subscription = $100 + x$.

No. of Subscribers = $1000 - 5x$.

$TR = (100+x)(1000-5x)$

$= 100000 - 500x + 1000x - 5x^2$

$= -5x^2 + 500x + 100000$.

$TR' = -10x + 500$.

For Max/Min ⇒ $TR' = 0$.

$-10x + 500 = 0$.

$x = 50$.

Rev. is max. when $x = 50$.

Exercise

(1) $C = \frac{1}{3}x^3 - 18x^2 + 160x$

$AC = \frac{1}{3}x^2 - 18x + 160$ $\quad MC = x^2 - 36x + 160.$

According to the question $AC = MC$

$$\frac{1}{3}x^2 - 18x + \cancel{160} = x^2 - 36x + \cancel{160}.$$

$$x^2 - \frac{1}{3}x^2 - 36x + 18x = 0.$$

$$\frac{2}{3}x^2 - 18x = 0.$$

$$2x\left(\frac{1}{3}x - 9\right) = 0.$$

$$x = 0 \quad \frac{1}{3}x - 9 = 0$$

$$x = 27.$$

$\therefore AC = MC$ when $x = 0$ or $x = 27$.

(2.) $C = 100 + 0.015x^2$ $\quad R = 3x.$

P = Profit = $R - C$

$P = 3x - 100 - 0.015x^2$

$P = -0.015x^2 + 3x - 100$

$P' = -0.03x + 3$

For Maxima/Minima $P' = 0 \Rightarrow -0.03x + 3 = 0.$

$$x = 100.$$

$P'' = -0.03 < 0$

$\therefore$ Profit is Maximum when $x = 100$

(3.) Demand: $x = 1200 - 60p$.

$$\frac{dx}{dp} = -60.$$

Elasticity of Demand: $\eta_d = -\frac{p}{x} \cdot \frac{dx}{dp}$

$$-\frac{p}{1200-60p} \times (-60) = 1. \quad \left[\begin{array}{l}\because \text{Demand is} \\ \text{Unit Elastic}\end{array}\right.$$

$$\frac{60p}{1200-60p} = 1$$

$$120p = 1200$$

$$p = 10.$$

$\therefore$ Demand is unit elastic when $p = 10$.

(4.) Demand Function: $x = 10 - p$. or $p = 10 - x$.

$\therefore AR \Rightarrow p = 10 - x. \Rightarrow TR = AR.x.$

$$= (10-x)x$$

$$= 10x - x^2$$

$$TC = \frac{x^2}{4}.$$

In monopoly, profit is max when

$$MR = MC.$$

$$TR' = TC'$$

$$10 - 2x = \frac{x}{2}$$

$$10 = \frac{x}{2} + 2x$$

$$10 = \frac{5}{2}x$$

$$x = 4$$

$\therefore$ Profit is max. when $x = 4$.

(5.) Let the no. of vacant rooms be x.

$\therefore$ No. of rented rooms $= 40 - x$.

Since an inc. of ₹100 results in 2 vacant rooms.

$\therefore$ Inc. of ₹50 results in 1 vacant room.

$\therefore$ Per day Rent $= 1000 + 50x$.

Daily Income $= R = (40-x)(1000+50x)$.

$$= 40000 + 2000x - 1000x - 50x^2$$

$$= 40000 + 1000x - 50x^2$$

$\cancel{R' = +1000 + 100x}$

$R' = 1000 - 100x$

For Maxima / Minima $R' = 0$

$1000 - 100x = 0 \Rightarrow x = 10$.

(6.) $p = 700 - 3x \Rightarrow TR = 700x - 3x^2$

$TC = 2x^2 + 250x + 2000 - 50x$

$\cancel{= 2x^2 + 300x + 2000}$

$TC = 2x^2 + 200x + 2000$.

Profit is Max when

$MR = MC$

$TR' = TC'$

$700 - 6x = 4x + 200 \Rightarrow 500 = 10x$

$\Rightarrow x = 50$

(7.) A

(8.) $y = 48x - 2x^2$

$\frac{dy}{dx} = 48 - 4x$

For Max/Min. $\Rightarrow \frac{dy}{dx} = 0 \Rightarrow 48 - 4x = 0$

$x = 12$

$\therefore y$ (TR) is max when $x = 12$

(9.) $TC = 100000 + 150x$ $\quad p = 200 \Rightarrow TR = 200x.$

At Break-even point

$TR = TC$

$200x = 100000 + 150x.$

$50x = 100000$

$x = 2000.$

(10.) $C = 2x^2 + x - 5$

$MC = C' = 4x + 1$

$MC(x=4) = 4 \times 4 + 1 = 17.$

(11.) $q = ap^{\alpha}$

$TR = pq = ap^{\alpha} . p = ap^{\alpha+1}$

$TR' = a(\alpha+1) . p^{\alpha}$

if $\alpha < -1 \Rightarrow \alpha + 1 < 0 \Rightarrow TR' < 0$

because TR' is a product of a, $\alpha+1$ and p^{α}

of these a and p^{α} are positive

but $\alpha+1$ is negative as explained above.

$\therefore$ The product i.e. TR' is negative.

if TR' is negative

$\Rightarrow$ TR is a decreasing function.

(12.) $C = 4860 + 750000q + 15q^2$

$$AC = \frac{4860}{q} + 750000 + 15q$$

$$AC' = -\frac{4860}{q^2} + 15$$

For Max or Min. $AC' = 0$

$$\frac{+4860}{q^2} = +15$$

$$q^2 = 324$$

$$q = 18.$$

$\therefore$ AC is min. when $q = 18$.

(13.) $C(x) = 0.008x^3 - 0.05x^2 - 20x + 5000$

The question needs to identify the function y

Two functions can be derived from $C(x)$

i.e. AC and MC.

$AC = 0.008x^2 - 0.05x - 20 + \frac{5000}{x}$ | $MC = 0.024x^2 - 0.1x - 2$

None of these matches the given function y.

$AC' = 0.016x - 0.05 - \frac{5000}{x^2}$ $MC' = 0.048x - 0.1$

y matches AC'.

$\therefore$ Ans C $\Rightarrow$ slope of AC or Marginal AC.

(14.) D (16) B.

(15.) C

Previous Years' Questions

(1.) $TC = a(x-5)^3 + b.$

$TC' = 3a(x-5)^2$ $TC'' = 6a(x-5)$

Let us first examine $TC' = 3a(x-5)^2$

Since $(x-5)^2 > 0$ for all x.

$\therefore TC' > 0$ for all x.

$\therefore$ TC is an increasing function.

Now let us examine TC''.

It is evident if $x-5>0$ $TC''>0$

i.e. $TC''>0$ when $x>5$

$\therefore$ TC is Concave Up when $x>5$

TC will be Concave Down when $x<5$

and thus $x=5$ will be the pt. of inflexion

$\therefore$ $\boxed{\text{Ans D}}$ TC is an inc. fn. (Upward sloping) with a point of inflexion.

(2.) Using the relation $MR = AR\left(1-\frac{1}{e}\right)$

We know $MR>0$ when $e>1$

when $MR>0$ TR is increasing.

Thus TR increases when $e>1$

$\boxed{\text{Ans B}}$

(3.) C (4) C

(5) $p = 300-5x. \Rightarrow TR = 300x-5x^2$

$MR = TR' = 300-10x$

$MR=0 \Rightarrow 300-10x=0$

$x=30.$

$\therefore p = 300-5\times30$

$= 150.$

(6.) $x = 200 - 6p^2$

$\frac{dx}{dp} = -12p.$

$\left.\frac{dx}{dp}\right]_{p=5} = -12\times5 = -60.$

$\eta_d = -\frac{P}{X} \cdot \frac{dx}{dp}$

$= -\frac{5}{200-6\times25} \times -60$

$= \frac{300}{200-150} = 6.$

(7.) $P = 45 - 5x$ $\quad C = 5x + 2x$

$C = 7x$

$TR = 45x - 5x^2$

Profit is Max when $MR = MC$

$TR' = TC'$

$45 - 10x = 7$

$x = 3.8.$

$\therefore P = 45 - 5\times3.8$

$= ₹26.$

(8.) $C = 0.1x^2 + 3 \Rightarrow MC = 0.2x$

(9.) $C = 0.005x^3 - 0.02x^2 - 30x + 5000$

$AC = 0.005x^2 - 0.02x - 30 + \frac{5000}{x}$

$MAC = AC' = 0.01x - 0.02 - \frac{5000}{x^2}$

(10.) Linear Demand function: $p = a + bx$

Case I: $x = 1000$ when $p = ₹4$

$\therefore \; 4 = a + 1000b$ — (1)

Case II: $x = 1500$ when $p = ₹2$

$\therefore \; 2 = a + 1500b.$ — (2)

Solving (1) & (2).

$-2 = 500b \Rightarrow b = \frac{-2}{500} = \frac{-1}{250}.$

from (1) $\Rightarrow 4 = a + 1000 \times \frac{-1}{250}.$

$4 = a - 4 \Rightarrow a = 8.$

$\therefore$ Demand Fn: $p = 8 - \frac{1}{250}x.$

$TR = px = 8x - \frac{1}{250}x^2$

$MR = 8 - \frac{2}{250}x$

$= 8 - \frac{1}{125}x.$

(11.) $p = 4 - 5x$.

$$\frac{dp}{dx} = -5 \Rightarrow \frac{dx}{dp} = -\frac{1}{5}.$$

$$\eta_d = -\frac{p}{x} \cdot \frac{dx}{dp} = 1$$

$$-\frac{(4-5x)}{x} \times -\frac{1}{5} = 1$$

$$\frac{4-5x}{5x} = 1 \Rightarrow 4 - 5x = 5x.$$

$$x = \frac{4}{10} = \frac{2}{5}.$$

(12) Av. Cost per repair = ₹320   No. of repairs $= 0.5t^{3/2}$

$\therefore$ Total Cost of repairs $= 320 \times 0.5t^{3/2}$
$= 160t^{3/2}$

Replacement Cost = ₹ 80000.

$\therefore$ Total cost of Car $= 80000 + 160t^{3/2}$

Optimum replacement period implies time when AC is minimum.

$$\therefore AC = \frac{80000 + 160t^{3/2}}{t} = \frac{80000}{t} + 160t^{1/2}$$

$$AC' = -\frac{80000}{t^2} + 80t^{-1/2}$$

For Max./Min $\Rightarrow AC' = 0$.

$$-\frac{80000}{t^2} + 80t^{-1/2} = 0.$$

$$\frac{80000}{t^2} = 80t^{-1/2}$$

$$1000 = t^{2-1/2}$$

$$1000 = t^{3/2}$$

$$t = (1000)^{2/3} = \left(1000^{1/3}\right)^2$$

$$= 10^2 = 100 \text{ months.}$$

(Since t is in months).

(13) Let the no. of units produced in excess of 500 be x.

Total Profit = Profit on 500 units + Profit on additional x units.

$$P = 500 \times 50 + x(50 - 0.1x)$$

$$P = 25000 + 50x - 0.1x^2$$

$$P' = 50 - 0.2x.$$

For Maxima / Minima $P' = 0 \Rightarrow x = 250$

$\therefore$ Level of output $= 500 + x$

$= 500 + 250$

$= 750$ units.

(14) It is known $MC = AC$.

$$C' = \frac{C}{x}.$$

$$C = x \cdot C'$$

$\therefore$ Total cost is a constant multiple of x.

(15.) $p = ae^{-kx}$

$\frac{dp}{dx} = a.e^{-kx} \times (-k)$ $\Rightarrow \frac{dx}{dp} = \frac{-1}{ake^{-kx}}$

$= -ake^{-kx}$

$\eta_d = -\frac{p}{x} \cdot \frac{dx}{dp}$

$= -\frac{\cancel{ae^{-kx}}}{x} \times \frac{-1}{\cancel{a}k\cancel{e^{-kx}}}.$

$= \frac{1}{kx}.$

(16.) $TC = \frac{1}{3}x^3 - x^2 + 5x + 2$

Here we need to evaluate the given options.

<u>option A</u> $TC' = x^2 - 2x + 5$

but $TC' \neq 0$ when $x = 3/2$

<u>option B</u> $AC = \frac{1}{3}x^2 - x + 5 + 2/x.$

$AC' = \frac{2}{3}x - 1 - \frac{2}{x^2}.$

when $x = \frac{3}{2}$ $AC' = \frac{2}{3} \times \frac{3}{2} - 1 - \frac{2}{(3/2)^2} \neq 0.$

<u>Option C</u> $\cancel{T}VC = \frac{1}{3}x^3 - x^2 + 5x.$

$AVC = \frac{1}{3}x^2 - x + 5$

$AVC' = \frac{2}{3}x - 1$

when $x = 3/2$ $AVC' = 0$ $\therefore$ AVC is min when $x = 3/2$

(17.) $C = x^3 - 315x^2 + 27000x + 20000.$

$C' = 3x^2 - 630x + 27000$

For Max / Min $\Rightarrow C' = 0$

$x^2 - 210x + 9000 = 0.$

$x^2 - 150x - 60x + 9000 = 0.$

$(x-150)(x-60) = 0$

$x = 60, 150.$

$\boxed{\text{Ans B.}}$

(18.) $C = 1500 + 30x + x^2$

$MC = C' = 30 + 2x.$

$MC\,(x=20) = 30 + 2\times 20 = 70.$

(19.) $p = \dfrac{e^x}{x}$

$\dfrac{dp}{dx} = \dfrac{x.e^x - 1.e^x}{x^2} = \dfrac{e^x(x-1)}{x^2} \Rightarrow \dfrac{dx}{dp} = \dfrac{x^2}{e^x(x-1)}$

$\eta_d = \dfrac{p}{x} \cdot \dfrac{dx}{dp}$

$= \dfrac{e^x/x}{x} \times \dfrac{x^2}{e^x(x-1)} = \dfrac{\cancel{e^x}}{\cancel{x^2}} \times \dfrac{\cancel{x^2}}{\cancel{e^x}(x-1)}$

$= \dfrac{1}{x-1}.$

(20.) $f(x) = e^x$

$f'(x) = e^x > 0$ for all $x \Rightarrow$ Increasing Function

$f''(x) = e^x > 0$ for all $x \Rightarrow$ Concave Up or Convex Down.

$\boxed{\text{Ans B}}$

(21.) c

(22.) $C = \frac{x^2}{4} + 3x + 400$

$AC^{0} = \frac{x}{4} + 3 + \frac{400}{x}$.

$AC' = \frac{1}{4} - \frac{400}{x^2}$

AC is Min when $AC' = 0$

$\frac{1}{4} - \frac{400}{x^2} \Rightarrow x^2 = 1600$

$x = 40$.

(23.) $e_1 > e_2 \Rightarrow$ demand in mkt 1 is more elastic

$\therefore$ price in mkt 1 must be less than that in mkt 2.

$\therefore P_1 < P_2$.

$\boxed{\text{Ans B}}$

(24.) c

(25.) See Solution of Exercise Q 11 (Pg 37)

(26.) $p = x e^{1/2}$

$\frac{dp}{dx} = e^{1/2} \Rightarrow \frac{dx}{dp} = \frac{1}{e^{1/2}}$

$\eta_d = -\frac{p}{x} \times \frac{dx}{dp}$

$= \frac{-\cancel{x}\cancel{e^{1/2}}}{\cancel{x}} \times \frac{1}{\cancel{e^{1/2}}} = -1.$

$\boxed{\text{Ans B}}$ for all val. of x demand is unit elastic for the given demand function.

(27.) See Exercise Q12 Pg 38.

(28.) $e_1 = \frac{11}{10} = 1.1$ $e_2 = \frac{27}{23} = 1.17.$

Since $e_1 < e_2$

$\therefore P_1 > P_2.$ $\boxed{\text{Ans A}}$.

(29.) A

(30.) See Exercise Q8 Pg 37.

(31.) $C = 2x^2 + x - 5 \Rightarrow MC = 4x + 1$

$MC(x=4) = 16 + 1 = 17.$

(32.) B

(33.) $y = 2x^3 - 15x^2 - 36x + 15$

$\frac{dy}{dx} = 6x^2 - 30x - 36$

for max / min $\Rightarrow \frac{dy}{dx} = 0$

$6x^2 - 30x - 36 = 0$

$x^2 - 5x - 6 = 0$.

$x^2 - 6x + x - 6 = 0$

$(x-6)(x+1) = 0$

$x = 6, -1$.

$\frac{d^2y}{dx^2} = 12x - 30$. for $x=6$ $\frac{d^2y}{dx^2} > 0$

for $x=-1$ $\frac{d^2y}{dx^2} < 0$.

$\therefore$ The fn is max. when $x=-1$.

(34.) $p = 100 - 0.2x \Rightarrow TR = px = 100x - 0.2x^2$

$C = 4x + 3000$.

Profit is maximum when

$MR = MC$

$TR' = TC'$

$100 - 0.4x = 4$

$96 = 0.4x$

$x = 240$

(35.) $f(x) = e^x \log x \, (2x^2+3)$

$f'(x) = (e^x)' \cdot \log x \cdot (2x^2+3) + e^x \cdot (\log x)' \cdot (2x^2+3)$

$+ e^x \cdot \log x \cdot (2x^2+3)'$

$= e^x \log x \, (2x^2+3) + e^x \cdot \frac{1}{x} \cdot (2x^2+3) + e^x \log x \, 4x.$

$= e^x \left(\log x \, (2x^2+3) + \frac{2x^2+3}{x} + \log x \cdot 4x \right).$

$= e^x \left(2x^2 \log x + 3\log x + 2x + \frac{3}{x} + 4x \log x \right).$

None of the given options is Correct.

(36.) $\frac{x}{x-y} = \log\left(\frac{a}{x-y}\right).$

$\frac{x}{x-y} = \log a - \log(x-y)$

$\frac{(x-y) \cdot 1 - x \cdot \left(1 - \frac{dy}{dx}\right)}{(x-y)^2} = 0 - \frac{1}{x-y} \times \left(1 - \frac{dy}{dx}\right).$

$\frac{\cancel{x} - y - \cancel{x} + x\frac{dy}{dx}}{(x-y)^2} = -\frac{1}{x-y} + \frac{1}{x-y}\frac{dy}{dx}.$

$$\frac{x\frac{dy}{dx}-y}{(x-y)^2} = \frac{-1}{(x-y)}\left(1-\frac{dy}{dx}\right)$$

$$x\frac{dy}{dx}-y = -(x-y)\left(1-\frac{dy}{dx}\right)$$

$$x\frac{dy}{dx}-y = (y-x)\left(1-\frac{dy}{dx}\right)$$

$$x\frac{dy}{dx}-y = (y-x) - (y-x)\frac{dy}{dx}.$$

$$\left(x+(y-x)\right)\frac{dy}{dx} = y-x+y.$$

$$y\frac{dy}{dx} = 2y-x.$$

$$\frac{dy}{dx} = \frac{2y-x}{y} = 2-\frac{x}{y}.$$

(37.) $p = 10e^{-x/400}.$

$TR = px = 10xe^{-x/400}.$

$MR = 10\left(x'.\,e^{-x/400} + x.\left(e^{-x/400}\right)'\right)$

$= 10\left(e^{-x/400} + x.\,e^{-x/400} \times \frac{-1}{400}\right).$

$$MR = 10e^{-x/400}\left(1 - \frac{x}{400}\right)$$

$$MR = 0$$

$$10\,e^{-x/400}\left(1 - \frac{x}{400}\right) = 0$$

$$1 - \frac{x}{400} = 0$$

$$1 = \frac{x}{400} \Rightarrow x = 400.$$